COMPUTATIONAL TECHNOLOGY FOR COMPARISON
OF TWO METHODOLOGIES FOR POSITION
FINDING SEARCH REGIONS

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THESIS

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FINDING SEARCH REGIONS

by

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of
Two Methodologies for Position Finding Search Regions

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ABSTRACT

This thesis considers two models for the computation of position finding confidence, one of which utilizes a bivariate normal distribution of region, and the other a chi-square distribution. The two models are based on different assumptions, these are explained and explored. A computer simulation model is presented which utilizes both position finding models under varying conditions.

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I. INTRODUCTION

Bearings are taken on an emitter (target) of unknown geographic location by n stations of known location. The best point estimate (BPE) of the target is then calculated, along with a confidence, or search, region within which the target may be said to be located with a given probability. This constitutes the classic radio direction finding position location problem.

This classic problem is part of a general class of position estimation problems whose basic approach is identical, which is the calculation of a point location from a number of lines of position.

Variant problems in this class occur when lines of position are established from ranges, or relative times of arrival of an emission, or when the object to be located takes the measurements on the reference stations.

While this paper examines the classic problem, much of the methodology applies to this more general class of problems.

The BPE is the primary result of a position location solution. The confidence region amplifies this information in two ways. The size of the confidence region serves as a measure of the reliability of the BPE, a larger region implying a more tenuous location. The region also describes an area in which the target will be located with specified confidence. The geometric details of this region provide the more likely locations for search.

II. COMPUTATION OF CONFIDENCE REGIONS

A typical [6] sequence of operations for a position estimation by computer is as follows: A first point estimate is made and a plane surface tangent to the earth is established at that point. Remaining computations are made in the plane rather than the surface of the earth. Unusable, or wild, bearings are rejected and a BPE made utilizing the remaining bearings. A confidence region is computed using the same bearings.

More than one mathematical method is available for most of the steps [3, 4, 8, 9, 11]. The two methods used herein for confidence region computation are designated the bivariate normal (BVN) and the chi-square (χ^2).

The Bivariate normal region [3, 4] is the most commonly used. This region is the family of concentric ellipses

$$ax^2 - 2bxy + cy^2 = -2 \log (1 - P)$$

where P = probability that the target is found within the ellipse

x, y = coordinates in local (to the target) reference system

$$a = \sum_{j=1}^{n} \frac{\cos^2\theta_j}{\sigma_j^2}$$

$$b = \sum_{j=1}^{n} \frac{\sin\theta_j \cos\theta_j}{\sigma_j^2}$$

$$c = \sum_{j=1}^{n} \frac{\sin^2\theta_j}{\sigma_j^2}$$

θ_j = bearing observed by station j

σ_j^2 = variance of displacement of j^{th} line of bearing

n = number of bearings

The bearings from the stations are assumed to be normally distributed with a mean of zero, and are displaced parallel to the true bearing at the site of the BPE. The confidence region lies in a plane tangent to the surface of the earth at the BPE as calculated by the least squares method [3, 4].

The BVN confidence region requires several assumptions:

(1) The position lines are great circles on the surface of the earth.

(2) Measurement error is Gaussian with mean zero.

(3) Displacement of position lines from the true position line is parallel to the true position line.

(4) The earth is flat in the vicinity of the confidence region.

These last two assumptions represent approximations that are made to simplify the calculation of the confidence region. Thus, the calculated region is an approximation. The parallel bearing displacement is an approximation of the actual displacement of a point on the bearing by a bearing error. From this, the probability distribution of the points is developed [4]. Placing the resultant curves in a plane adds an additional distortion, which grows with the size of the confidence region [7].

The chi-square confidence region [3, 10] is the family of concentric curves

$$X^2 = \sum_{j=1}^{n} \frac{1}{S_j^2} \cdot (\Theta_j - \beta_j)^2$$

where

S_j^2 = bearing variance of station j

θ_j = bearing observed by station j

β_j = true bearing from station j to a point (x,y) in the curve

n = number of bearings

X^2 has a chi square distribution with n degrees of freedom. Of the above listed assumptions required by the BVN method, only (1) and (2) are required for the χ^2 region. The region lies on the surface of the earth centered at the minimum X^2 point, rather than in a plane.

Both methods require the estimated variance of the bearings to be used in the calculation. The effect of an incorrect estimation could be much larger than the errors of approximation. Estimates in practice are a function of a stations past performance plus the direction finder operator's evaluation of the reliability of each bearing he observes and reports.

The assumption of Gaussian bearing distribution eliminates the case of wild, or outlying bearings which occur in practice, largely through operator error. As a Gaussian distribution

is basic to both the BVN and χ^2 methods, elimination of wild bearings from the computation is necessary, though the theory of each method would be affected by their assumed existence. Wild bearing rejection methods are available [6, 7]. The use of such methods is usually limited to four or more bearing situations. With two bearings, no basis for detection of wild bearings exists. For most three bearing situations, when the existence of at least one wild bearing is indicated, it is impossible to identify it, or them. With four bearings, one wild bearing can be seen to miss the BPE area formed by the other three.

In practice [6, 7] the Outlier detection and rejection method is used, but a Gaussian distribution is still assumed for those remaining.

Daniels and Vajda [3] object to the use of χ^2 regions on the grounds that it may occur that no points can be found to satisfy the resultant equations.

Read [10] examines the simplifications required in calculating the BVN regions, and compares the two methods. The χ^2 regions are expected to be too small, and the BVN too large, though the full effect of the simplifications by Kukes and Starik [4] is not known.

Comparison of the two methods by computer plotting techniques shows considerable difference in size. Additionally, the effect of the approximations used in the BVN method is apparent. While BVN region is elliptical in all cases, the χ^2 region may assume an egg shape.

A barrier to the use of χ^2 regions is the lack of a way of reporting them. A BVN region can be described by its center, angular orientation, and length of semi axes. No such simplifying parameters exist for χ^2 regions. One possible approach would be to attempt a conversion into a partially equivalent rectangle, square, or circle as is often done for BVN regions [1, 6, 7]. Since these regions are always larger than the computed region, contain areas of lesser probability, and omit areas of greater probability, any advantage of using one method over the other would probably be lost thereby.

A thorough investigation of the two methods is required, in order to determine the actual probabilities represented by each type of confidence region. The differences indicate that at least one of the methods has error. A computer simulation was written to compare them.

III. COMPUTER SIMULATION MODEL

The appended computer program provides a means for simulation of the two confidence region methods for identical input data.

Inputs to the program are station latitude, longitude, and standard deviation of bearing distribution; target latitude and longitude.

The true bearings and distance for each station and target are computed and a bearing error generated and added. This portion contains parts of the Bergh program in Pope [8] and is a program used by LT Paul Winkler, modified here to include more general positions.

A first-point estimate of the position is made by Pope's [9] vector method, and the final fix by the least squares method of Kukes and Starik [4] in a plane tangent to the earth at the first-point estimate. This portion was used by Lunde [5] but contained errors since corrected.

A bivariate normal confidence region is computed by the method of Kukes and Starik, and orientation angle and length of major and minor simi axes generated. Additionally, output by graphic plot is available.

A χ^2 confidence region is then computed and a graphic plot generated. No parameteric output for the χ^2 region is available, as no method yet exists for describing the shape of the region. This portion was originally written by LT Winkler, and since modified to include more general positions.

The subroutines GAMA, for the normalized incomplete gamma function, and MTRMAP, the graphic plot, are from the library of the W. R. Church Computer Center at the Naval Postgraduate School.

The graphic plots are on a grid measured in degrees of a great circle, resembling a Mercator projection at the equator. Hence, visual distortion of the regions occurs as their location approaches the poles, but the geographical points of the χ^2 regions are correct. The geographical points of the BVN regions are correct, except for the distortion caused by the flat earth assumption.

IV. SUGGESTED SIMULATION PROCEDURE

An undetected error in previous work invalidated all data generated. Time constraints prevented regeneration of the data after correction of the error.

The behavior of the regions generated by the two methods differs with the number of stations in the net, configuration of the net, standard deviation of the error used, and relative location of target with respect to the net. Thus, a general case may not exist. A valid comparison would include sufficient combinations to detect all possible patterns of behavior.

A useful measure of effectiveness for the regions would be the difference in the probability level in each of the two regions occupied by the true position of the target.

The standard deviation used in generating bearing error is also used in computing BPE and confidence regions. A minor modification to the program would permit the examination of the consequences of incorrectly estimating the actual bearing distribution of a station, by computing these with a different standard deviation that that used to generate the error.

The errors generated are from a normal distribution, and include no wild bearing. A means of detection for wild bearings is not included in the program. Such a routine may be added to the program, along with a generation routine for wild bearings.

The modifications above would permit examination of more realistic situations through the use of operational data, along with a model of the actual net which supplied the data.

V. AN ALTERNATE APPROACH

An underlying assumption in the methods discussed is that there is a uniform prior probability for the target location. That is, the mathematical solution of the BPE and confidence regions are inputs to the solution of the operational question of target location. Prior knowledge of probable target location does not affect the solution. This approach is intuitively correct generally.

Operationally, there are two uses of position finding information:

(1) To determine the location of a target of known or unknown identity;

(2) To identify a target from a set of known targets by its location.

This second use introduces prior information into the problem, and occurs usually when attempting to identify a fixed land-based station.

Butterly [2] describes a method for incorporation of information of this type in producing a Bayesian position estimate, though he implies that its use could be more general.

A further modification of the program would permit a test of the efficacy of this method by comparing the number of correct identifications when using the Bayesian method with the number when selecting the target in the lowest level of the confidence region. Both the BVN and χ^2 methods could be used in each.

```
C
C     THIS FORTRAN PROGRAM CALCULATES RADIO DIRECTION FINDING POSITION
C     ESTIMATES AND CONFIDENCE REGIONS. SEVERAL OPTIONS IN OUTPUT ARE
C     AVAILABLE, AND ARE DOCUMENTED BY COMMENT CARDS IN THE PROGRAM.
C
C     INPUTS TO THE PROGRAM ARE:
C     1 - NUMBER OF DIRECTION FINDING STATIONS (ONE CARD, FORMAT I2)
C     2 - LATITUDE, LONGITUDE, AND STANDARD DEVIATION OF EACH STATION
C         (ONE CARD EACH, FORMAT 3F10.4)
C     3 - NUMBER OF TARGETS (ONE CARD, FORMAT I2)
C     4 - LATITUDE AND LONGITUDE OF EACH TARGET (ONE CARD EACH,
C         FORMAT 2F10.4)
C     5 - SIZE OF GRAPHICAL PLOTS IN DEGREES (ONE CARD, FORMAT F10.4)
C
C     LIMITS - STATIONS 8
C              TARGETS  8
C
C     SIGN CONVENTION: LATITUDES   NORTH  0 TO +90.0
C                                  SOUTH  0 TO -90.0
C                      LONGITUDES  EAST   0 TO -180.0
C                                  WEST   0 TO +180.0
C                      BEARINGS    EAST OF NORTH 0 TO +180.0
C                                  WEST OF NORTH 0 TO -180.0
C
C
C
C     PART ONE - INITIALIZATION AND INPUT
C
      DIMENSION BETA(10,10)
      DIMENSION STEEST(8,8), GAMMA(8,8)
       DIMENSION STALAT(10), STALON(10), STDEV(10), TARLAT(20)
      DIMENSION TARLON( 20), THETA( 10,20), DIST( 10,20)
      DIMENSION WORM(13,13)
      DIMENSION PJ(10),ZETA(10)
      DIMENSION B1(8), B2(8)
      DIMENSION AE(8),BE(8),CE(8),DE(8),EE(8),FE(8)
      DIMENSION Y(13,13)
      DIMENSION SDRAD(8)
      DIMENSION C(13,13)
      REAL*4 T(24)
C
      RADDEG=.017453293
      IX=46721
      PI=3.141592
      RADIUS=10800.0/PI
      EX = 0.0
C
C
C     READ STATION POINTS
```

```
C
      READ(5,100) NSTA
  100 FORMAT(I2)
      DO 2 I=1,NSTA
    2 READ (5,102) STALAT(I), STALON(I), STDEV(I)
  102 FORMAT(3F10.4)
C
C
C     READ TARGET POINTS
C
      READ(5,100) NTAR
      DO 1 I=1,NTAR
    1 READ(5,102) TARLAT(I), TARLON(I)
C
C
C     READ GRAPH SIZE
C
      READ(5,102) CGRAPH
C
C
C
C
C     PART TWO - GENERATION OF BEARINGS AND BEARING VARIANCES
C
C
C
C
C
C     THIS SECTION OF PART TWO COMPUTES BEARING ANGLES AND DISTANCES
C     USING NAPIER'S ANALOGIES
C
      DO 10 I=1,NSTA
      DO 10 J=1,NTAR
      AA=ABS(STALON(I)-TARLON(J))
      IF(AA.GT.180.0) AA=360.0-AA
      GAMMA(I,J)=AA
C
C     CHECK FOR SAME MERIDIAN CASE
C
      IF(AA.GT.0.01) GO TO 105
      BB=0.0
      IF(STALAT(I).GT.TARLAT(J)) BB=180.0
      THETA(I,J)=BB
      DIST(I,J)=ABS(STALAT(I)-TARLAT(J))*RADIUS*RADDEG
      GO TO 23
C
  105 CONTINUE
C
      CC=ABS(AA-180.0)
```

```
      IF(CC.GT.0.001) GO TO 106
      DD=0.0
      SUMLAT=STALAT(I)+TARLAT(J)
      IF(SUMLAT.LT.0.0) DD=180.0
      THETA(I,J)=DD
      EED=180.0-ABS(SUMLAT)
      DIST(I,J)=EED*RADIUS*RADDEG
      GO TO 23
  106 CONTINUE
C
C   CONTINUE SOLUTION OF TRIANGLE
C
      WVALUE = GAMMA(I,J)*.5*RADDEG
      COT1 = COS(WVALUE)/SIN(WVALUE)
      BRAVO = 90.0 - TARLAT(J)
      ABLE = 90.0 - STALAT(I)
C
      COS1 = COS((ABLE - BRAVO)*.5*RADDEG)
      COS2 = COS((ABLE + BRAVO)*.5*RADDEG)
      SIN1 = SIN((ABLE - BRAVO)*.5*RADDEG)
      SIN2 = SIN((ABLE + BRAVO)*.5*RADDEG)
C
      YA=COT1*COS1
      YB=COT1*SIN1
C
      ATA=ATAN2(YA,COS2)
      ATB=ATAN2(YB,SIN2)
      BBB=(ATA-ATB)/RADDEG
C
C
C   BBB=ANGLE, NOW COMPUTE PROPER SIGN
C
      SL=STALON(I)
      TL=TARLON(J)
      IF(SL.LT.0.0) SL=360.0+SL
      IF(TL.LT.0.0) TL=360.0+TL
      TL=TL-SL
      IF(TL.LT.0.0) TL=TL+360.0
      IF(TL.LT.180.0) BBB=-BBB
      THETA(I,J)=BBB
C
C   THETA=TRUE BEARING FROM STATION TO TARGET
C
C
C
C
C   DISTANCE FROM STATION TO TARGET USING LAW OF COSINES
C
      GAMMA(I,J) = GAMMA(I,J)*RADDEG
```

```
      TARLAT(J)=TARLAT(J)*RADDEG
      STALAT(I)=STALAT(I)*RADDEG
      TEMP = SIN(TARLAT(J))*SIN(STALAT(I))+COS(TARLAT(J))*COS(STALAT(I))
     1*COS(GAMMA(I,J))
      DFDIST = ARCOS(TEMP)
      DIST(I,J) = DFDIST*RADIUS
C
C  DIST=DISTANCE IN NAUTICAL MILES
C
      TARLAT(J)=TARLAT(J)/RADDEG
      STALAT(I)=STALAT(I)/RADDEG
C
   23 CONTINUE
C
C
C
C
C  THIS SECTION OF PART TWO ADJUSTS THE STANDARD DEVIATION TO BE USED
C  IN BEARING ERROR COMPUTATION FOR THE DISTANCE BETWEEN STATION AND
C  TARGET.  THE SOURCE OF THIS COMPUTATION IS NOT KNOWN.
C
      SD = STDEV(I)
      R = DIST(I,J)/100.0
      IF (R-54.0) 20,20,22
   22 SD = SD* (R/(108.0 - R))
      GO TO 32
   20 IF (R-10.0) 24,26,26
   24 IF (R-4.0) 28,30,30
   28 SD = PI/2.0
      GO TO 32
   30 SD = SD*(.0204*R*R - .402*R + 3.0)
      GO TO 32
   26 SD =SD*(.00071*R*R - .0213*R + 1.1598)
   32 STEEST(I,J) = SD*SD*RADDEG*RADDEG
   10 CONTINUE
C
      WRITE(6,253)
  253 FORMAT (///,36X,45H BEARING ANGLE FROM ITH STATION TO JTH TARGET,/
     1/)
      WRITE(6,256)((I,J,THETA(I,J),J=1,NTAR),I=1,NSTA)
  256 FORMAT(39X,13HFROM STATION ,I2,
     111H TO TARGET ,I2,4H -- ,F10.4)
C
      WRITE(6,255)
  255 FORMAT (//,40X,40HDISTANCES FROM ITH STATION TO JTH TARGET,//)
      WRITE(6,256)((I,J,DIST(I,J),J=1,NTAR),I=1,NSTA)
C
C
```

```
C   THIS SECTION OF PART TWO COMPUTES BEARING ERROR FROM A SIMULATED
C   NORMAL DISTRIBUTION USING A RANDOM NUMBER GENERATOR,AND ADDS THE
C   ERROR TO THE TRUE BEARING.
C
      WRITE(6,260)
  260 FORMAT (//,50X,15HNORMAL VARIATES,//)
C
      DO 40 I=1,NSTA
      DO 40 J=1,NTAR
      STEEST(I,J) = STEEST(I,J)/RADDEG
      CALL GASS(IX,STEEST(I,J),EX,X)
      BETA(I,J)=THETA(I,J)+X
C
      WRITE(6,256)I,J,X
C
   40 CONTINUE
C
C
C   OUTPUT ANGLES WITH ASSOCIATED ERROR
C
      WRITE(6,200)
  200 FORMAT (//6X,6HSTALAT, 8X, 6HSTALON, 8X,6HTARLAT, 8X,6HTARLON,
     18X,10HTHETA(I,J), 4X, 5HSTDEV)
      WRITE(6,259)((STALAT(I),STALON(I),TARLAT(J),TARLON(J),
     1 BETA(I,J),STEEST(I,J), J=1,NTAR), I=1,NSTA)
  259 FORMAT(/,6(F12.5,2X))
C
C
C
C
C   PART THREE - COMPUTATION OF POSITION ESTIMATE, OR FIX
C
C
C
      DO 3321 J1=1,NTAR
C
      WRITE(6,220) TARLAT(J1), TARLON(J1)
  220 FORMAT(71H1        THE FOLLOWING BEARINGS WERE TAKEN ON A TARGET LOCA
     1TED AT LATITUDE, F12.5,11H LONGITUDE,F12.5,//)
      WRITE(6,300)
  300 FORMAT(10X,8HLATITUDE,12X,9HLONGITUDE,14X,5HTHETA,11X,12HTRUE BEAR
     1ING, 10X, 7HSTD DEV,/)
      WRITE(6,221) ( STALAT(I),STALON(I), BETA(I,J1),THETA(I,J1),
     1STDEV(I) ,I=1,NSTA)
  221 FORMAT(5(10X,F10.5))
C
C
C
```

```
C  THIS SECTION OF PART THREE CALCULATES A FIRST POINT ESTIMATE OF
C  THE FIX USING POPE'S VECTOR METHOD (1971)
C
C
C  COMPUTE BEARING PLANE INTERSECTIONS
C
      WRITE(6,701)
  701 FORMAT(//,44X,26HBEARING LINE INTERSECTIONS,
     1/,40X,8HSTATIONS,4X,8HLATITUDE,4X,
     29HLONGITUDE,//)
      DO 88 I=1,NSTA
C
      B1(I) = STALAT(I) * RADDEG
      B2(I) = STALON(I) * RADDEG
C
      SLAT = SIN(B1(I))
      CLAT = COS(B1(I))
      SLON =-SIN(B2(I))
      CLON = COS(B2(I))
      SBNG = SIN(BETA(I,J1)*RADDEG)
      CBNG = COS(BETA(I,J1)*RADDEG)
C
      XE = CLAT * CLON
      YE = CLAT * SLON
      ZE = SLAT
C
      AE(I) = CBNG*SLON - SBNG*SLAT*CLON
      BE(I) = -CBNG*CLON - SBNG*SLAT*SLON
      CE(I) = SBNG*CLAT
C
      DE(I) = BE(I)*ZE - CE(I)*YE
      EE(I) = CE(I)*XE - AE(I)*ZE
      FE(I) = AE(I)*YE - BE(I)*XE
C
   88 CONTINUE
C
      USUM = 0.0
      VSUM = 0.0
      WSUM = 0.0
C
      DO 89 I=2,NSTA
      ILIMIT = I-1
      DO 90 J=1,ILIMIT
C
      UE = BE(I)*CE(J) - CE(I)*BE(J)
      VE = CE(I)*AE(J) - AE(I)*CE(J)
      WE = AE(I)*BE(J) - BE(I)*AE(J)
C
```

```
      TI = UE*DE(I) + VE*EE(I) + WE*FE(I)
      TJ = UE*DE(J) + VE*EE(J) + WE*FE(J)
C
      IF (TI*TJ .LT. 0.0) GO TO 90
      IF (TI .GT. 0.0) GO TO 91
C
      UE = -UE
      VE = -VE
      WE = -WE
C
C
C  COMPUTE CENTROID OF INTERSECTIONS
C
   91 USUM = USUM + UE
      VSUM = VSUM + VE
      WSUM = WSUM + WE
C
      UMAG=SQRT(UE*UE+VE*VE+WE*WE)
      ULAT=ARSIN(WE/UMAG)/RADDEG
      ULON=-ATAN2(VE,UE)/RADDEG
C
      WRITE(6,702)I,J,ULAT,ULON
  702 FORMAT(40X,I2,5H AND ,I2,3X,F8.3,4X,F8.3)
C
C  ULAT, ULON = BEARING PLANE INTERSECTIONS
C
   90 CONTINUE
   89 CONTINUE
C
C
C  CONVERT CENTROID TO LATITUDE AND LONGITUDE OF FIRST POINT ESTIMATE
C
      X1LON =-ATAN2(VSUM,USUM)/RADDEG
      X1LAT=ARSIN(WSUM/SQRT(USUM*USUM+VSUM*VSUM+WSUM*WSUM))/RADDEG
C
      WRITE(6,222)
      WRITE(6,223) X1LAT, X1LON
  222 FORMAT(//,10X,24HTHE FIRST POINT ESTIMATE,//)
  223 FORMAT(10X,8HX1LAT = ,F10.5,5X,8HX1LON = ,F10.5,//)
C
C
C
C  THIS SECTION OF PART THREE TRANSFERS THE PROBLEM TO A PLANE TANGENT
C  TO THE EARTH AT THE FIRST POINT ESTIMATE, THEN COMPUTES A POSITION
C  ESTIMATE USING THE LEAST SQUARES METHOD OF KUKES AND STARIK.
C
      ASUM=0.0
      BSUM=0.0
```

```
      CSUM=0.0
      DSUM=0.0
      ESUM=0.0
C
      DO 8999 L=1,NSTA
      SDRAD(L)=STDEV(L)*RADDEG
 8999 CONTINUE
      DO 9002 I=1,NSTA
      XSI=BETA(I,J1)*RADDEG
      XSI=ABS(XSI)
C
C   XSI=STATION BEARING IN RADIANS
C
C
C   COMPUTE BEARING IN LOCAL REFERENCE PLANE USING NAPIER'S ANALOGIES
C   AND LAW OF SINES
C
      RHO=(STALON(I)-X1LON      )*RADDEG
      RHO=ABS(RHO)
      IF(RHO.GT.PI) RHO=(2.0*PI)-RHO
C
C   CHECK FOR DUE NORTH OR SOUTH BEARING
C
      IF(RHO.GT.0.001) GO TO 345
      ZETA(I)=PI
      IF(STALAT(I).LT.X1LAT) ZETA(I)=0.0
      GO TO 347
  345 CONTINUE
      RR=ABS(RHO-PI)
      IF(RR.GT.0.001) GO TO 346
      ZETA(I)=PI
      POLE=STALAT(I)+X1LAT
      IF(POLE.LT.0.0) ZETA(I)=0.0
      GO TO 347
C
  346 CONTINUE
C
C   CONTINUE SOLUTION OF TRIANGLE
C
      CEE=(90.-STALAT(I))*RADDEG
      DENOM=.5*(XSI+RHO)
      RUM11=.5*(XSI-RHO)
      RUM12=.5*CEE
C
      ANUM=SIN(RUM11)*TAN(RUM12)
      DENA=SIN(DENOM)
      ARKTN1=ATAN2(ANUM,DENA)
      RTA=ABS(DENOM-PI/2)
```

```
      RTB=ABS(RUM11-PI/2)
      IF(RTA.GT.0.001) GO TO 355
      BNUM=COS(RUM11)*TAN(RUM12)
      DENB=0.001
      GO TO 357
  355 IF(RTB.GT.0.001) GO TO 356
      BNUM=0.001
      DENB=COS(DENOM)
      GO TO 357
  356 CONTINUE
      BNUM=COS(RUM11)*TAN(RUM12)
      DENB=COS(DENOM)
  357 CONTINUE
      ARKTN2=ATAN2(BNUM,DENB)
      AAEE=ARKTN1+ARKTN2
      IF(XSI.LT.0.001) XSI=0.001
      DDD=SIN(CEE)*SIN(XSI)
      IF(AAEE.LT.0.001) AAEE=0.001
      EEE=SIN(AAEE)
      FFF=DDD/EEE
      IF(FFF.GE.1.0) FFF=0.999999
      CCEE=ARSIN(FFF)
C
      ZETA(I)=PI-CCEE
      IF(AAEE.LT.CEE) ZETA(I)=CCEE
      IF(BETA(I,J1).LT.0.0) ZETA(I)=-ZETA(I)
C
  347 CONTINUE
C
C  ZETA=LOCALIZED BEARING
C
C
C  TAKE NEGATIVE OF ZETA TO CONVERT TO NORMAL TRIG SIGN CONVENTION
C
      ZETA(I)=-ZETA(I)
C
C
C  CALCULATE PERPENDICULAR DISTANCE FROM LOCALIZED BEARING TO
C  CENTER OF COORDINATE SYSTEM
C
      YPRIM=(90.-XILAT      )*RADDEG
      AAEE=ABS(AAEE)
      YDIST=    (YPRIM-AAEE)
      PJ(I)=YDIST*SIN(ZETA(I))
C
C
C  SUM TERMS FOR FOLLOWING CALCULATIONS
C
```

```
      ASUM=ASUM+(COS(ZETA(I))*COS(ZETA(I)))/(SDRAD(I)*SDRAD(I))
      BSUM=BSUM+(SIN(ZETA(I))*COS(ZETA(I)))/(SDRAD(I)*SDRAD(I))
      CSUM=CSUM+(SIN(ZETA(I))*SIN(ZETA(I)))/(SDRAD(I)*SDRAD(I))
C
 9002 CONTINUE
C
      DO 9006 K=1,NSTA
      DSUM=DSUM+PJ(K)*((BSUM*SIN(ZETA(K))-CSUM*COS(ZETA(K)))/(SDRAD(K)*
     1SDRAD(K)))
      ESUM=ESUM+PJ(K)*((ASUM*SIN(ZETA(K))-BSUM*COS(ZETA(K)))/(SDRAD(K)*
     1SDRAD(K)))
C
 9006 CONTINUE
C
C
C   CALCULATE LEAST SQUARES ESTIMATES AND CONVERT TO DEGREES
C
      COEF=1.0/((ASUM*CSUM)-(BSUM*BSUM))
      XO=COEF*DSUM
      YO=COEF*ESUM
      SQKO=-2.*ALOG(.1)
      XEST=X1LON+XO/RADDEG
      YEST=X1LAT+YO/RADDEG
C
C   XEST, YEST = LEAST SQUARES ESTIMATE OF POSITION
C
C
      WRITE(6,9007) YEST,XEST
 9007 FORMAT(' ','THE LEAST SQUARES ESTIMATE OF POSITION IS ',F6.2,
     1'   DEGREES LATITUDE'//43X,F7.2,'  DEGREES LONGITUDE'///)
C
C
C
C
C
C   PART FOUR - COMPUTATION OF CONFIDENCE REGIONS
C
C
C
C
C   THIS SECTION OF PART FOUR CALCULATES A BIVARIATE NORMAL CONFIDENCE
C   REGION (DANIELS, KUKES AND STARIK)
C
C
C
C   CALCULATE ANGLE BETWEEN MAJOR AXIS OF ELLIPSE AND MERIDIAN
C
      AA1=2.*BSUM
      AA2=CSUM-ASUM
      TWOANG=ATAN2(AA1,AA2)
```

```
      BANG=TWOANG/2.0
C
      IF(BANG)651,651,652
  651 BANG=BANG+(PI/2.0)
      GO TO 653
  652 BANG=BANG-(PI/2.0)
  653 CONTINUE
      ANG=BANG/RADDEG
C
C
C
C   CALCULATE SEMIAXES FOR ELLIPSE OF .9 PROBABILITY
C
      IF(BANG.GT.1.56) BANG=1.56
      IF(BANG.LT.-1.56) BANG=-1.56
C
      SDENOM=CSUM+BSUM*TAN(BANG)
      FALSE=ABS(SDENOM)
      SAMAJO=SQRT(SQKO/FALSE)
      SDENO=ASUM-BSUM*TAN(BANG)
      FATS=ABS(SDENO)
C
      SAMINO=SQRT(SQKO/FATS)
      SAMINO=SAMINO/RADDEG
      SAMAJO=SAMAJO/RADDEG
C
C
C
      WRITE(6,9081)  ANG,SAMAJO,SAMINO
 9081 FORMAT(' ',10X,'ANGLE BETWEEN MAJOR AXIS AND MERIDIAN = ',F6.2//
     134X,'SEMI-MAJOR AXIS = ',F6.2,//,34X,'SEMI-MINOR AXIS = ',F6.2///)
C
C
C
C
C   THIS SECTION OF PART FOUR PRINTS A GRAPHIC PLOT OF A BIVARIATE
C   NORMAL CONFIDENCE REGION.  THE SIZE IS DETERMINED BY THE INPUT
C   'CGRAPH'.  USE THE SMALLEST VALUE OF CGRAPH THAT WILL DISPLAY
C   THE REGION, TO MINIMIZE DISTORTION.
C
C   THIS SECTION MAY BE SKIPPED IF THIS OUTPUT IS NOT DESIRED.  JUMP
C   FROM THIS POINT TO THE BEGINNING OF THE CHI-SQUARE SECTION, BELOW.
C
C   LOAD PROBABILITY ARRAY FOR BIVARIATE NORMAL
C
      XOR=XEST
      YOR=YEST
      DO 9011 J=1,13
```

```
      EXVAL=(CGRAPH/2.0)-(((J-1)/12.0)*CGRAPH)
      EXVAL=EXVAL*RADDEG
      DO 9012 I=1,13
      YVAL=(CGRAPH/2.0)-(((I-1)/12.0)*CGRAPH)
      YVAL=YVAL*RADDEG
      HOO=BSUM*EXVAL*YVAL-(ASUM/2.)*EXVAL*EXVAL-(CSUM/2.0)*YVAL*YVAL
      PROB=1.0-EXP(HOO)
      WORM(I,J)=PROB
 9012 CONTINUE
 9011 CONTINUE
C
      N=13
      M=13
      AZ=0.0
      BZ=0.0
      BND=0.0
      AMIN=0.0
      IJT=0.0
      ICON=1
      IGR=1
      DO 9080 LL=1,24
 9080 T(LL)=0.0
C
C
C  PRINT THE CONFIDENCE REGION PLOT
C
      CALL MTRMAP(WORM,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON,IGR)
      WRITE(6,591)
  591 FORMAT(///,45X,'BIVARIATE NORMAL CONFIDENCE REGION')
      WRITE(6,509) CGRAPH,CGRAPH
  509 FORMAT(///,20X,37H  THE SIZE OF THE CONFIDENCE PLOT IS ,F10.4,
     114H DEG. LAT. BY ,F10.4, 12H DEG. LONG. ,/,40X,
     241H  THE FIX POINT IS LOCATED AT (007, 007). )
C
C
C
C  THIS SECTION OF PART FOUR CALCULATES A CHI-SQUARE CONFIDENCE
C  REGION, AND PRINTS A GRAPHICAL PLOT.  NO OTHER OUTPUT IS AVAILABLE.
C  ADDITIONALLY, LEVELS OF CONSTANT VALUE OF Q MAY BE PLOTTED (DANIELS).
C  THIS SECTION MAY BE SKIPPED BY JUMPING FROM THIS POINT TO THE END OF
C  THE MAIN PROGRAM.  EITHER THE Q OR CHI-SQUARE PLOTS MAY BE SKIPPED
C  SEPARATELY BY JUMPING AROUND THE APPROPRIATE 'CALL MTRMAP...' CARD
C  AT THE END OF THIS SECTION.
C
      X1LAT=YEST
      X1LON=XEST
      X1LAT = X1LAT + CGRAPH/2.0
      X1LON = X1LON + CGRAPH/2.0
```

```
C
      DO 501 K = 1,13
      DO 502 L = 1,13
      Y(K,L) = 0.0
C
      DO 571 I=1,NSTA
      G=ABS(STALON(I)-X1LON)
      IF(G.GT.180.0) G=360.0-G
      IF(G.GT.0.001) GO TO 610
      ANGLE=0.0
      IF(STALAT(I).GT.X1LAT) ANGLE=180.0
      GO TO 620
  610 CONTINUE
      ABLE=90.0-STALAT(I)
      BRAVO=90.0-X1LAT
      WVALUE=G*.5*RADDEG
      COT1 = COS(WVALUE)/SIN(WVALUE)
      COS1 = COS((ABLE-BRAVO)*.5*RADDEG)
      COS2 = COS((ABLE+BRAVO)*.5*RADDEG)
      SIN1 = SIN((ABLE-BRAVO)*.5*RADDEG)
      SIN2 = SIN((ABLE+BRAVO)*.5*RADDEG)
      RR=COT1*SIN1
      SS=ATAN2(RR,SIN2)
      TT=COT1*COS1
      UU=ATAN2(TT,COS2)
      ANGLE=(UU-SS)/RADDEG
  620 CONTINUE
      SL=STALON(I)
      TL=X1LON
      IF(SL.LT.0.0) SL=360.0+SL
      IF(TL.LT.0.0) TL=360.0+TL
      TL=TL-SL
      IF(TL.LT.0.0) TL=TL+360.0
      IF(TL.LT.180.0) ANGLE=-ANGLE
      CHIDIF=ANGLE-BETA(I,J1)
      Y(K,L) = Y(K,L) + CHIDIF*CHIDIF/STDEV(I)/STDEV(I)
      PAR1=NSTA/2.
      PAR2=Y(K,L)/2.
      CALL GAMA (PAR1,PAR2,GAM,B,0.0)
      C(K,L)=1.-GAM
  571 CONTINUE
C
  502 X1LON = X1LON - CGRAPH/12.0
      X1LON = X1LON + CGRAPH*13.0/12.0
  501 X1LAT = X1LAT - CGRAPH/12.0
C
C
C  SET UP THE CONTOUR SUBROUTINE
```

```
C
      N=13
      M = 13
      AZ = 0.0
      BZ = 0.0
      BND = 0.0
      AMIN = 0.0
      IJT = 0
      ICON = 1
      IGR = 1
      DO 504 II = 1,24
  504 T(II) = 0.0
C
C  PLOT THE Q LEVELS
C
      CALL MTRMAP(Y,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON,IGR)
      WRITE(6,592)
  592 FORMAT(///,45X,'LEVELS OF CONSTANT VALUE OF Q')
      WRITE(6,509) CGRAPH,CGRAPH
C
C  PLOT THE CHI-SQUARE REGION
C
      CALL MTRMAP(C,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON,IGR)
      WRITE(6,593)
  593 FORMAT(///,45X,'CHI-SQUARE CONFIDENCE REGION')
      WRITE(6,509) CGRAPH,CGRAPH
C
 3321 CONTINUE
C
      STOP
      END


      SUBROUTINE GASS(IX,S,AM,V)
      A = 0.0
      DO 50 I=1,12
      CALL WRANDU(IX,IY,Y)
      IX = IY
   50 A = A+Y
      V = (A-6.0)*S+AM
      RETURN
      END
```

```
      SUBROUTINE WRANDU(IX,IY,Y)
      IY = IX*65539
      IF (IY) 5,6,6
    5 IY = IY + 2147483647+1
    6 Y = IY
      Y = Y*.4656613E-9
      RETURN
      END
```

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13. ABSTRACT

This thesis considers two models for the computation of position finding confidence, one of which utilizes a bivariate normal distribution of region, and the other a chi-square distribution. The two models are based on different assumptions, these are explained and explored. A computer simulation model is presented which utilizes both position finding models under varying conditions.

KEY WORDS	LINK A		LINK B		LINK C	
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Position Finding Program						
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